# Particle Swarm Optimization Based HMM Parameter Estimation for Spectrum Sensing in Cognitive Radio System

**Yogesh Vineetha, E. S. Gopi and Shaik Mahammad**

**Abstract** Spectrum Estimation has emerged as the major bottleneck for the development of advanced technologies (IoT and 5G) that demand for a unperturbed continuous availability of the spectrum resources. Opportunistic dynamic access of spectrum by unlicensed users when the licensed user is not using the resources is seen as a solution to the pressing issue of spectrum scarcity. The idea proposed for spectrum estimation is to model the Cognitive Radio (CR) network as Hidden Markov Model (HMM). The spectral estimation is done once in a frame. 100 such frames with 3000 slots each is considered for performing the experiment, assuming that the PU activity is known for a fraction of 3.33% of the slots i.e., for 100 slots. The parameters of the HMM are estimated by maximizing the generating probability of the sequence using the Particle Swarm Optimization (PSO). For the typical values of the network parameters, the experiments are performed and the results are presented. A novel sum squared error minimization based "Empirical Match" algorithm is proposed for an improved latent sequence estimation.

**Keywords** Computational intelligence · Particle swarm optimization · Empirical match algorithm · Cognitive radio · Spectrum estimation · Hidden Markov model

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# 1 Introduction

A fast pace development of wireless technologies such as Internet Of Things and the emerging 5G technology demand for a continuous availability of spectrum resources for all its users. However, the major bottleneck is the spectrum scarcity problem that might be a consequence of either poor throughput of the network due to congestion or lack of available spectrum resources [5, 8, 9]. The Cognitive Radio technology aims at improving the spectrum utilization and network throughput by enabling the unlicensed users (Secondary Users (SU)) to access the resources of the licensed user (Primary User (PU)) whenever the PU is not utilizing the allocated resources. For an extensive utilization of the resources, the SU must learn the behavior or trend of how the PU is utilizing the resources. Physical spectrum sensing based on energy detection is performed in-order to know the presence of licensed user. However owing to the erroneous channel conditions due to external noise and interference from other users, it is highly possible that the results of physical spectrum sensing are not reliable. The following cases may arise:

1. If the PU is idle, and the physical spectrum sensing decides that the PU is active - **False alarm**, then it will lead to under-utilization of the spectrum resources, since neither the PU is using the channel nor the SU can make use of the free channel owing to false detection. This will have an adverse impact on the throughput and efficiency of the channel.
2. If the PU is busy and the SU senses the channel to be free - **Miss detection**, then the PU and the SU will simultaneously attempt to transmit data i.e., channel contention thereby resulting in congestion.

Hence, the SU cannot rely on the outcome of physical spectrum sensing. A more accurate estimate of PU activity is needed for opportunistic spectrum access by the SU to utilize the spectrum resources without leading to congestion in network and also increasing the network throughput.

# 2 Problem Formulation

The SU performs the physical spectrum sensing at sensing slots that are uniformly distributed over time. Let the PU spectrum access be represented by the random vector $\overline{\mathbf{S}}$. The outcome of the random vector in each sensing slot can be either 0 (PU is inactive) or 1 (PU is active). The SU performs spectrum sensing based on energy detection. Let $\overline{V}$ be a random vector that denotes the outcome of physical spectrum sensing, the outcome of the random vector will be a binary sequence. Under ideal conditions the outcome $\overline{V}$ is same as $\overline{S}$. However, in real time the result of the physical spectrum sensing and the actual PU activity are seldom in coherence. Hence, the problem under consideration is to estimate the PU activity as accurately as possible, given the SU observation sequence. The solution to this problem is to model the CR network as HMM [1] and extract the hidden network parameters which will further aid in estimating the PU activity more precisely.

## 2.1 Hidden Markov Model

The Hidden Markov Models (HMM) belongs to the class of mixture models, where the latent variables are discrete and belong to a finite set. The HMM consists of two stochastic processes of which one is hidden - latent. The other stochastic process is a result of the hidden process and is referred to as observation sequence. The hidden process in case of HMM is a markov process. A markov process of first order can be defined as the one in which the next state of the process depends only on the current state of the process and is independent of all the past history of the process. As mentioned earlier, the latent states of the HMM form the markov chain of first order [2, 3].

The latent state of the HMM at $n$th instant can be represented by a variable $S_n$ that takes a value from a discrete set of $k$ values and yields an observation $V_n$, where, k is the number of distinct states involved in the markov process. The latent process of the HMM is characterized with the transition probability defined as $p_{ij} = P(s_{n+1} = j/s_n = i)$, where $i, j \in k$. The observation process is a result of the hidden latent process that is generated as a result of emission of observations from the latent states. The emission of the observations depends on the emission probabilities defined as $h_{ij} = P(v_n = j/s_n = i)$. Each possible value of latent state is associated with a prior probability $\pi_k = P(s_1 = k)$, such that, $\sum_k \pi_k = 1$. The transition probabilities and emission probabilities of a two state HMM can be represented in the form of tables as in Fig. 1. In the Fig. 1, $S_i$ and $S_a$ are the two distinct states in the markov process, where subscript $'i'$ and $'a'$ indicate inactive and active in accordance with the application in CR network. Similarly, $V_i$ and $V_a$ implies the observation result, where, i and a hold the same meaning.

A HMM can be completely described by a model defined by $\lambda = [\pi \quad A \quad O]$, where, $A$ is a matrix representing all possible transition probabilities between different states and O represents all the possible emission probabilities.

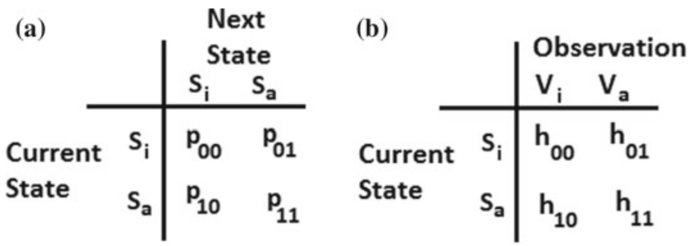


**Fig. 1** **a** Transition probability table. **b** Observation probability table

Regarding the problem of spectrum estimation, the only information available with the SU is the erroneous observation sequence $\overline{V}$. The HMM model corresponding to the CR Spectrum estimation problem is as shown in Fig. 2. Using $\overline{V}$, the HMM model (i.e., $\lambda = [\pi \quad A \quad O]$) is to be estimated. Using the estimated HMM model parameters, further, the PU activity sequence is to be estimated. Hence, the Spectrum Estimation problem in hand can be broadly split into two tasks, where the solution of first problem paves way to solve the second problem.

1. **Task 1** To estimate the model $\lambda$
2. **Task 2** To estimate the hidden PU activity $\overline{S}$.

A generalized flow chart illustrating the steps employed for solving spectrum estimation problem is as in Fig. 3.

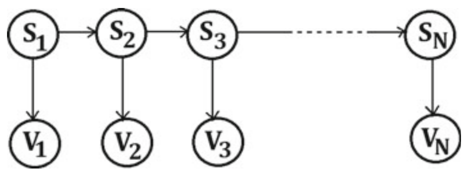


**Fig. 2** Hidden Markov model of SU in cognitive radio network

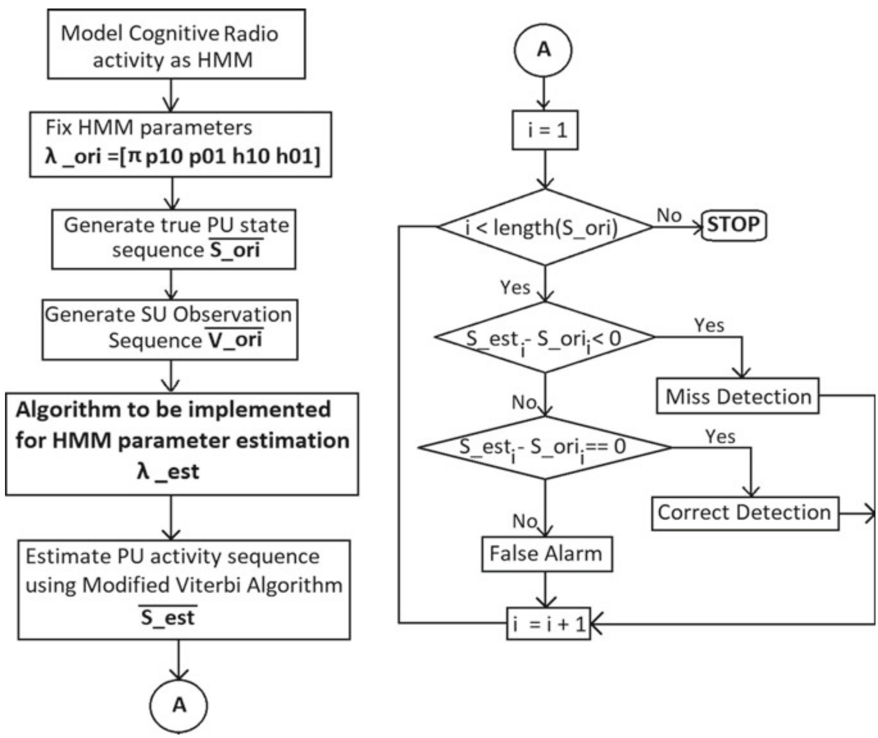


**Fig. 3** Generalized steps involved in spectrum estimation problem

# 3 PSO Based Estimation of Hidden Parameters of Network - Task 1

The first problem associated with the spectrum estimation is to estimate the hidden parameters of the network (CR modeled as HMM). This can be solved conventionally by using Expectation Maximization (EM) algorithm. The algorithm aims at finding the solution that maximizes the probability of generation of model.

## 3.1 Expectation Maximization

The EM algorithm [4] is an iterative algorithm that tries to adjust the model parameters such that the probability of generation of the random vector $\overline{V}$, given the model parameters increases. It provides the Maximum likelihood solution to the problem. The generation probability of the model can be described by (1)

$$G = (\pi_0)^{1-s_1}(1-\pi_0)^{s_1}\prod_{i=1}^{i=N-1} p_{s_i s_{i+1}} h_{s_{i+1} v_{i+1}} \tag{1}$$

The EM algorithm can be summarized as follows

1. Initialize the model with random parameters
2. Run EM algorithm (Refer Appendix) till convergence (200 iterations)
3. Repeat steps 1 and 2 for 10 times
4. From output in step 3, choose the best solution (model parameters that give maximum probability of generation) as $\lambda_{est}$.

However, there are a few drawbacks associated with the EM algorithm

1. The EM algorithm takes large time for convergence, i.e., slow convergence
2. The solution obtained using EM depends on the initialization, a bad initialization will result in convergence of EM algorithm to a local maxima.

## 3.2 PSO Based Proposed Technique

The use of CI technique - Particle Swarm Optimization (PSO) can be used to overcome the disadvantages of conventional EM algorithm. Particle Swarm Optimization (PSO) is a biologically inspired computational intelligence algorithm. Swarm here refers to a group of living objects such as a flock of birds, school of fishes etc. Each bird in the flock is technically referred to as particle in the swarm. The aim of the algorithm is to emulate the biological behavior of birds in the way they inter-communicate to reach their home(destination). It has been studied that the final decision of the bird about the direction in which it has to fly is based on the individual decision of that bird (Local Decision) and the decision of the flock (Global Decision). The objective is to minimize the distance of the bird from their current location to their final destination. Hence in general, PSO is a minimization algorithm. Each particle in the swarm is a potential solution to the problem [6, 7].

**PSO Objective Function**

The PSO is modeled to find the solution to HMM that maximizes the probability of generation of the model. The probability of generation of the model is formulated in terms of two iteratively updated variables. A forward probability variable $F(r, t)$ is considered, which takes care of generation of the random sequence $\overline{V}$, upto the $r$th frame, with the condition that the $r$th frame gets generated from $t$th state. The forward probability variable can be written in a recursive manner as,

- Initialization: $F(1, 0) = \pi_0 * P(V_1/S_1 = 0)$ and $F(1, 1) = \pi_1 * P(V_1/S_1 = 1)$
- Recursive equation

$$\begin{aligned} F(r, t = i) = F(r-1, t = 0) \times p_{0,i} \times P(V_r/t = i) + \\ F(r-1, t = 1) \times p_{1,i} \times P(V_r/t = i); \quad \forall i = 1 \ or \ 0 \end{aligned} \tag{2}$$

- Terminate when r = N (final state/ final slot)

A backward probability vector $B(r, t)$ is used, which governs the generation of random vector $\overline{V}$ from $(r + 1)$th frame to $N$th frame, under the condition that the $r$th frame gets generated from $t$th state. Backward probability can be written in a recursive manner as

- Initialization: $B(N, 0) = 1$ and $B(N, 1) = 1$
- Recursive equation

$$\begin{aligned} B(r, t = i) = B(r+1, t = 0) \times p_{i,0} \times P(V_{r+1}/t = 0) + \\ B(r+1, t = 1) \times p_{i,1} \times P(V_{r+1}/t = 1); \quad \forall i = 1 \ or \ 0 \end{aligned} \tag{3}$$

- Terminate when $r = 1$.

The generation probability of HMM can be written in terms of $F(r, t)$ and $B(r, t)$ as

$$G = \sum_{t=0}^{1} F(r, t) \times B(r, t); r \in 1, 2, 3, \ldots, N \tag{4}$$

The objective function of PSO is hence formulated using (4) as $J = \frac{1}{G}$.

**PSO Algorithm**

The outcome of the vector $[\pi_0 \ p_{10} \ p_{01} \ h_{10} \ h_{01}]$ (hidden parameters) is treated as the position of the particle. The distance from the destination position is treated as the objective function $J = \frac{1}{G}$. It is noted that the elements of the vector are probabilities and hence the range is restricted between 0 to 1. The PSO algorithm is adopted that minimizes the objective function $J$ is as given below.

1. Initialize the positions of the particles $a_1, a_2, \ldots a_N$. (with the elements of the vector ranging from 0 to 1).
2. initialize the tentative next positions of the birds $b_1, b_2, \ldots b_N$ (with the elements of the vector ranging from 0 to 1). Compute the corresponding cost function associated with the corresponding particles as $J_1, J_2, \ldots J_N$.
3. Compute $t = arg_i min J_i$.
4. Identify the next set of locations as follows.
   $c_i = |a_i + \alpha_1 \times (b_i - a_i) + \alpha_2 \times (b_t - a_i)|$
   if ($c_i >= 1$), $c_i$ is randomly chosen with the elements ranging from 0 to 1.
5. Assign $a_i = c_i$.
6. Compute the cost function associated with the corresponding particles $c_1, c_2, \ldots c_N$ as $K_1, K_2, \ldots K_N$.
7. If $J_i > K_i$, then $b_i = c_i$ else $b_i = b_i$.
8. Repeat the steps for finite number of iterations and the best particle's position corresponding to the lowest functional value $J$ is declared as the estimated hidden parameter.

An illustration of how particles move in PSO occurs is as in Fig. 4. In the figure, the boundary of box is having range from $-1$ to $+1$, because, the particle elements are probabilities that can take a valid value in range 0 to 1. 3 particles are considered.

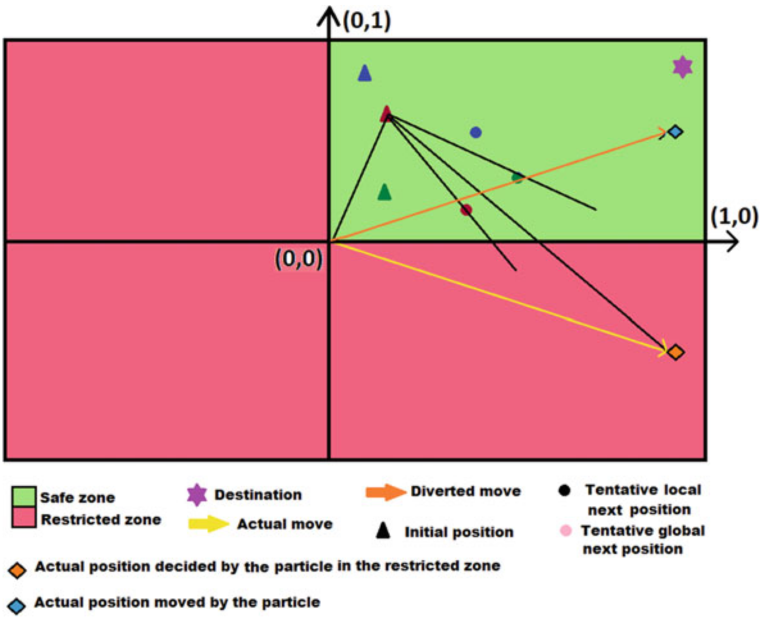


**Fig. 4** Illustration of PSO particle movement

In the illustration, the triangles represent the initial position of the particles and the circles represent the tentative next position (local decision) of the particles. The global decision of the flock (green circle) can be considered as the one that is nearest to the destination (purple star). For understanding the particle motion in space, let us consider the maroon color particle (triangle) movement. The particle movement is governed by a linear combination of the particle's local decision (maroon circle) and the flock global decision (green circle). Also, in the box, the portion colored green is the safe zone which corresponds to a valid probability value and red color corresponds to the restricted zone - invalid probability values. A case of the linear combination resulting in the location of the particle's next position (orange diamond) in the restricted zone is considered for illustration. In such case, the particle is flipped back to a position in the safe zone (blue diamond). A case that the particle position outside the box boundary might also arise. In such a case, the particle is positioned in a random location within the safe zone and iterations continue.

Since it is assumed that the PU activity is known for first 100 slots out of 3000 slots in a frame, the information is utilized to estimate the parameters of the HMM parameters. The initialization of PSO is made using the known information.

## 4 Proposed Technique to Estimate the Outcome of Random Vectors - Task 2

### Empirical Match Algorithm

Estimation of the the outcome of the random vector $\overline{\mathbf{S}}$ given the outcome for the random vector $\overline{\mathbf{V}}$ and the estimated hidden parameters $\lambda_{est}$ from Task 1 is proposed to be done using the Empirical Match algorithm as described below. For the estimation, it is assumed that the actual outcome of the random vector $\overline{\mathbf{S}}$ is known for $1/30$th (3.33%) of the sequence. The rest of the 96.67% of the sequence is estimated using the empirical Match algorithm. The objective of the algorithm is to minimize the Sum Squared Error (SSE) of the stochastic parameters used for estimating $\overline{S}$. The algorithm of the proposed technique is as follows

1. Trend of transitions in the sequence is assumed to be known for $1/30$th of the sequence (say n slots information is known)
2. For the next outcome of random vector $\overline{V}$, i.e., $V_{n+1}$, the SSE is calculated considering the possibility of generation of $V_{n+1}$ from $S_{n+1} = 0$ and $S_{n+1} = 1$.
3. The $S_{n+1}$ that gives lower value of SSE is considered and the estimated sequence $\overline{S_{est}}$ is updated.
4. Repeat steps 2 and 3 till $n = N$.

The flow chart of the algorithm is as in Fig. 5. The algorithm tries to track closely the changes in the parameters while estimating the activity of the PU in every successive slot, there by providing a reliable estimate of the PU activity.

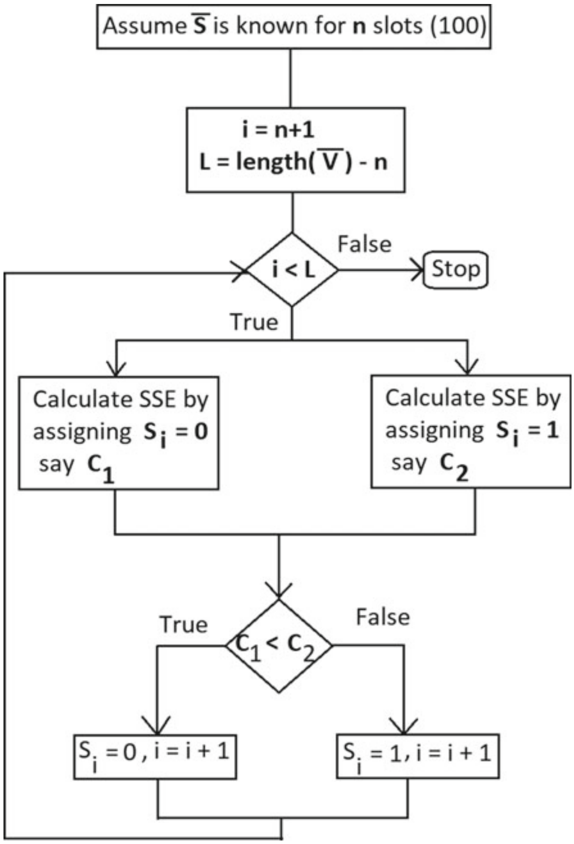


**Fig. 5** Flowchart illustrating the empirical match algorithm

## 5 Experimental Setup and Results

The spectrum sensing is performed at uniform intervals of times referred to as sensing slots or just slots. The sensing results (1 or 0 based whether the PU is active or inactive in the slot respectively) in the slots are considered as the outcome of random vector $\overline{S}$. It is assumed that the CR network parameters donot cange over 300000 slots (Stationary process). For the purpose of experiment, the spectrum sensing data is arranged into a 100 $\times$ 3000 matrix. Each row of the matrix is referred to as a frame. It is assumed that activity of the PU is known for 3.33% of slots per frame, i.e., 100 slots per frame. HMM model of the CR network is considered and the model parameters are the set of transition and emission probabilities of the HMM model represented by the vector $\lambda = [\pi, p_{10}, p_{01}, h_{10}, h_{01}]$. The definition of the elements of the HMM model is provided in Sect. 2. The elements of vector $\lambda$ reflects to the PU activity $-[\pi, p_{10}, p_{01}]$ and the erroneous channel conditions $-[h_{10}, h_{01}]$. The PU is assumed not to change its state within a slot and also it is assumed that the probability that the PU continues to be active/inactive for some slots continuously, once it becomes active/inactive is high i.e., $p_{11}$ *and* $p_{00}$ is high compared to $p_{10}$ *and* $p_{01}$. The choice of the emission probabilities $h_{01}$ *and* $h_{10}$ is made randomly, a high value of which indicates bad channel conditions and hence a high probability of error in sensed sequence and vice-versa.

**Estimation of PU Activity Using Conventional and Proposed Techniques**

The spectrum estimation is performed using

1. Conventional method Expectation Maximization followed by Empirical match algorithm
2. Proposed method using PSO followed by Empirical match algorithm.

Experiments were performed for 6 different sets of typical combinations of CR network parameters (HMM parameters). The performance of the proposed algorithms is compared with the conventional solution. The result is oriented on estimating the PU activity. The solution obtained using the Proposed CI technique and the conventional method is compared with the result of the physical spectrum sensing which clearly illustrates the need for the proposed algorithm. The comparison is done in terms of percentage of match of the estimated sequence with the actual PU activity sequence. Also, the percentage of miss detection and false alarm is compared which is an indication of reduced number of errors in the estimated sequence. The spectrum estimation was performed using the conventional EM algorithm and the PSO algorithm followed by empirical match algorithm and results are tabulated in Table 1.

The PSO algorithm with 500 particles was run for 10 iterations and the convergence graph is as shown in Fig. 6. Also an illustration of the HMM parameter estimation using PSO is shown in Fig. 7 for one set of typical network parameters (refer to set 1 in Table 1). The solution obtained using PSO is compared with the original parameters as well as with the initial estimate of parameters using the known

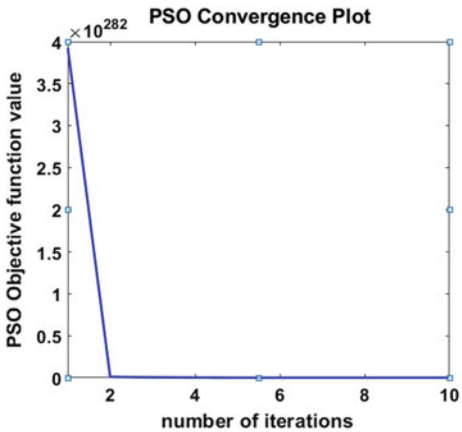


**Fig. 6** PSO Convergence Plot

**Table 1** Comparison of performance of conventional algorithm and proposed algorithm with the physical spectrum sensing result in estimating the PU activity for different sets of typical CR network parameters

| Parameter sets | True network parameters | | | | | Observations | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\beta_0$ | $p_{10}$ | $p_{01}$ | $h_{10}$ | $h_{01}$ | Match | | | Miss detection | | False alarm | |
| | | | | | | Physical sensing | Without CI (using EM) | With CI (using PSO) | Without CI (using EM) | With CI (using PSO) | Without CI (using EM) | With CI (using PSO) |
| Set 1 | 0.4900 | 0.2000 | 0.2200 | 0.8500 | 0.7700 | **18.8817** | 24.0630 | **81.4850** | 39.4520 | **7.8963** | 36.4850 | **10.6187** |
| Set 2 | 0.2200 | 0.1600 | 0.2500 | 0.7800 | 0.8500 | **19.1513** | 61.6260 | **80.9130** | 21.1910 | **10.4267** | 17.1830 | **8.6603** |
| Set 3 | 0.6200 | 0.2000 | 0.1200 | 0.5100 | 0.9000 | **24.8067** | 40.5517 | **75.9867** | 0.6023 | **17.9533** | 58.8460 | **6.0600** |
| Set 4 | 0.1800 | 0.2000 | 0.2300 | 0.6600 | 0.8900 | **23.3490** | 56.8863 | **75.4627** | 17.2067 | **13.6070** | 25.9070 | **10.9303** |
| Set 5 | 0.6200 | 0.2000 | 0.1200 | 0.5100 | 0.9000 | **24.7680** | 62.1423 | **75.6580** | 35.6203 | **18.1743** | 2.2373 | **6.1677** |
| Set 6 | 0.6000 | 0.1000 | 0.1500 | 0.8100 | 0.7400 | **21.8277** | 48.2133 | **75.4773** | 27.9520 | **15.0710** | 23.8347 | **9.4517** |

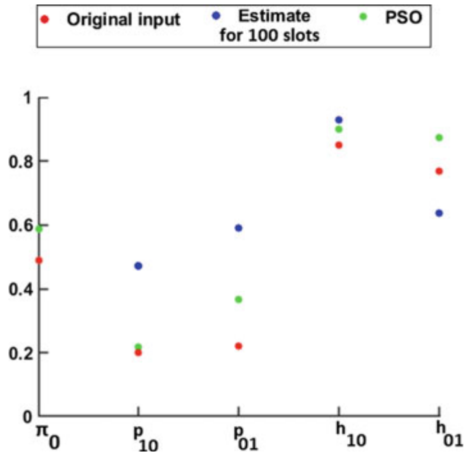


**Fig. 7** Parameter estimation using PSO (set 1 in Table 1)

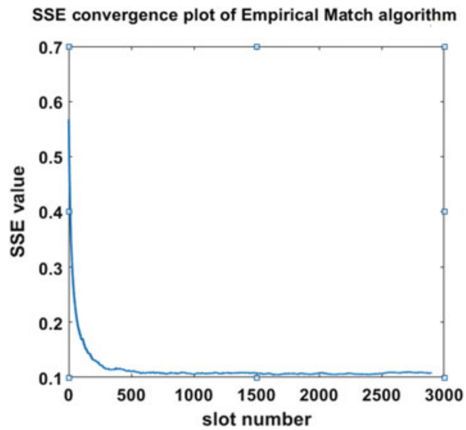


**Fig. 8** Convergence plot of empirical match algorithm (SSE minimization)

information (100 slots). It can be seen that, for all parameters, the PSO solution tries to converge closer to the original parameter value (Fig. 7).

The Empirical match algorithm is executed once the parameters are estimated using the PSO algorithm and Expectation maximization. The assumption that the PU activity is known for 3.33% of slots holds true for empirical match algorithm. Hence, using Empirical match algorithm, the rest 2900 slot activity of PU is estimated. Estimation of PU activity in every slot beginning from the 101th slot to the 3000th slot is done with the objective of reducing the Sum Square Error (SSE) between the estimated and obtained parameters. The Convergence plot of SSE in Empirical Match algorithm is as shown in Fig. 8.

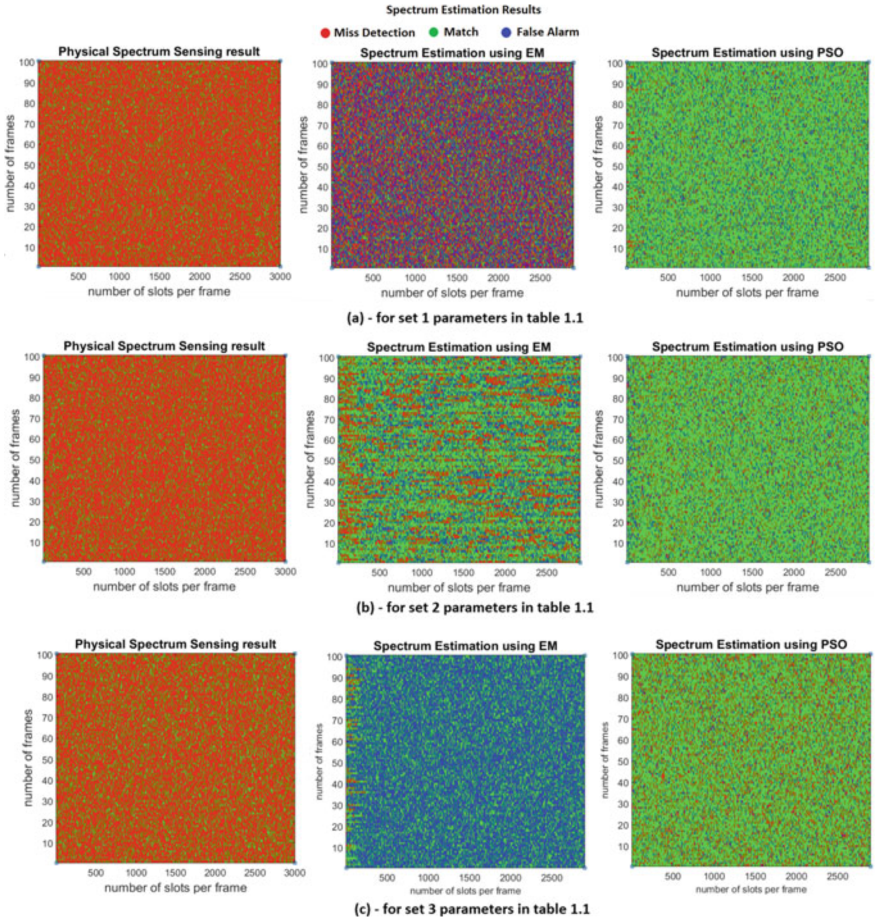


**Fig. 9** Illustration of comparison of physical spectrum sensing with the spectrum estimation using EM and PSO

An illustration of the results of physical spectrum sensing, estimation using EM followed by empirical match and estimation using PSO followed by empirical match is shown in Figs. 9 and 10. The figures correspond to the various sets of network parameters considered for performing the experiments. The figures follow a color code RGB, where, Red and Blue are used to represent the mismatch between the estimated sequence and the actual PU activity sequence. Amongst Red and Blue. Red indicates miss detection and Blue indicates false alarm. The green color is used to indicate the match which is the main focus of the experiments.

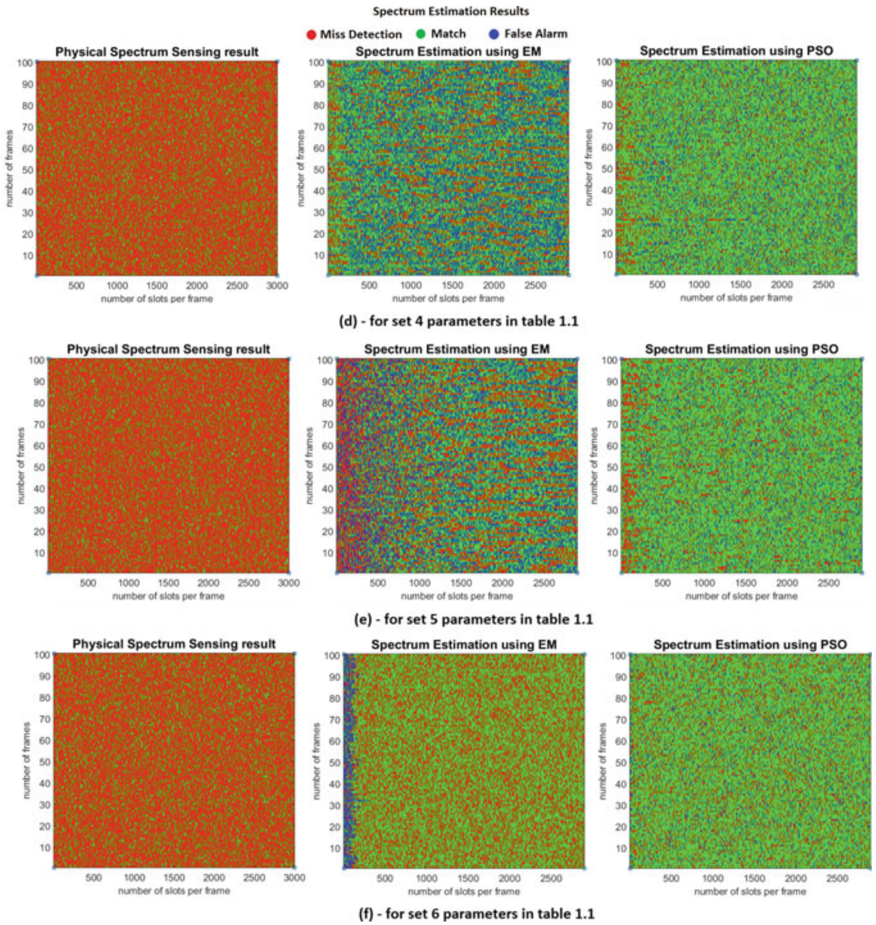


**Fig. 10** Illustration of comparison of physical spectrum sensing with the spectrum estimation using EM and PSO

## 6 Conclusion

The chapter proposes a computational intelligence based solution for spectrum estimation in cognitive radio networks. The solution proposed uses the Particle Swarm Optimization (PSO) followed by the novel Empirical Match algorithm. The futility of the physical spectrum sensing for opportunistic spectrum access can be overcome by using the proposed CI based technique. An average improvement of 55.36% over the physical spectrum sensing is obtained by the use of PSO followed by empirical match which accounts for additional match of one lakh sixty six thousand slots. This implies that the SU can better utilize the spectrum, thereby improving the spectrum utilization and network throughput. The proposed CI based algorithm was compared with the conventional solution to the problem using the expectation maximization followed by empirical match algorithm. It can be seen from Table 1 that the CI based solution to spectrum estimation problem outperforms the convention solution to problem as well as the physical spectrum sensing method (Energy Detection).

## 7 Future Scope

The experiments are performed based on the assumption that the PU activity is known for 3.33% of total time i.e., 100 slots. Scope for further reduction in the amount of known information can be seen. It is believed that the spectral estimation match can be improved by increasing the number of states in the HMM.

## 8 Appendix

**Expectation Maximization Algorithm** The EM algorithm used in our algorithm is as follows.

Consider the HMM with N observations. Let the observation sequence be defined as $\overline{V} = \left[V_1 V_2 \ldots V_r V_{r+1} \ldots V_N\right]$, where $r$ is the index variable, $r = 1, 2, \ldots N$.

Also, since the Latent states of the HMM are governed by the PU activity, the Latent state can be in either of the two states, let the state of the PU be denoted by a binary variable, $'t'$.

Let $F(r, t)$ denote the forward probability variable and $B(r, t)$ denote the backward probability variable.

F(r,t) means the probability of generating the observation sequence till the $r$th bit, with the condition that the $r$th bit is generated from PU being in $t$th state. A recursive formula for forward probability can be written as in Eq. 5. The initializations being: $F(1, 0) = \pi_0 * P(V_1/S_1 = 0)$ and $F(1, 1) = \pi_1 * P(V_1/S_1 = 1)$

$$\begin{aligned} F(r, t = i) = F(r - 1, t = 0) \times p_{0,i} \times P(V_r/t = i) + \\ F(r - 1, t = 1) \times p_{1,i} \times P(V_r/t = i); \quad \forall i = 1 \; or \; 0 \end{aligned} \tag{5}$$

Similarly, $B(r, t)$ denote the probability of generating the observation sequence from $(r + 1)$th bit till end, with the condition that the $r$th bit is generated from the PU being in state $t$. A recursive relation can be developed for finding the backward probability as in (6). The initializations are $B(N, 0) = 1$ and $B(N, 1) = 1$

$$\begin{aligned} B(r, t = i) = B(r + 1, t = 0) \times p_{i,0} \times P(V_{r+1}/t = 0) + \\ B(r + 1, t = 1) \times p_{i,1} \times P(V_{r+1}/t = 1); \quad \forall i = 1 \; or \; 0 \end{aligned} \tag{6}$$

Let $M(V_r, t = i)$ denote the fraction of $V_r$th bit being generated by the PU in state $t = i$, i∈(1,0 ). Then $M(V_r, t)$ can be obtained as

$$M(V_r, t = i) = \frac{F(V_r, t = i) \times B(V_r, t = i)}{F(V_r, t = 0) \times B(V_r, t = 0) + F(V_r, t = 1) \times B(V_r, t = 1)} \quad (7)$$

Let $Q(V_r, i, j)$ denote the fraction of bit $V_r$ being generated as a result of transition of PU from state i to state j

$$Q(V_r, t = i) = \frac{F(V_r, i) \times p_{ij} \times P(V_{r+1}/t = j) \times B(V_{r+1}, j)}{\sum_{i=0}^{1} \sum_{j=0,1} F(V_r, i) \times p_{ij} \times P(V_{r+1}/t = j) \times B(V_{r+1}, j)} \quad (8)$$

The HMM model parameters can be estimated to maximize the probability of generation of the observation given the stochastic HMM parameters. The different stochastic parameters namely, the transition probability, observation probability and the prior probability can be derived using (7) and (8).

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