Phase response of different commonly used systems

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1 Phase Response:

Four different types of systems are simulated and the phase response of the individual system and the cascade of the systems is plotted. N set of four different complex numbers $(x_{1i}, x_{2i}, x_{3i}, x_{4i})$ of magnitude less than unity are generated, which will be used as a pole or zero for these four types of systems. For every addition of complex pole or zero, a phase shift of 2π $(360\ degrees)$ can be observed. Systems and their phase response is explained below.

2 Different Systems:

2.1 System I:

Consider the system having a transfer function

$$H_{1i}(Z) = Z - X_{1i},\ where\ X_{1i} = \frac{r_{1i} + j r_{r2i}}{\sqrt{r_{1i}^2 + r_{r2i}^2}}\ and\ |X_{1i}| < 1$$

For this system, the phase will be $\phi = tan^{-1}\left(\frac{sin(\omega)+r_2}{cos(\omega)+r_1}\right)$ and at $\omega = sin^{-1}(r_2)$ the phase will be zero and around this frequency the phase response of the system takes a transition of 2π

An all zero system can be formed by cascading 'K' systems of transfer function H_{1i} i.e. $A_K = \prod_{i=1}^{K} H_{1i}(Z)$. As this system has 'K' zeros, a total of 'K' transitions can be observed.

Phase response of the individual subsystems H_{1i} and the system A_K are shown in figure 1 of the newsletter.

2.2 System II:

Consider a system having the transfer function

$$H_{2i}(Z) = \frac{1}{Z - X_{2i}},\ where\ X_{2i} = \frac{r_{3i} + j r_{r4i}}{\sqrt{r_{3i}^2 + r_{r4i}^2}}\ and\ |X_{2i}| < 1$$

As this system is having a pole, similar to system I, it's phase response also makes a transition of π. When 'K' such systems are cascaded, it forms an all pole system with transition function $B_k = \prod_{i=1}^{K} H_{2i}(Z)$. As B_K has 'K' poles, a total of 'K' transitions can be observed.

Phase response of the individual subsystems H_{2i} and the system B_K are shown in figure 2 of the newsletter.

2.3 System III:

Consider a system having the transfer function

$$H_{3i}\left(Z\right) = Z^{-1} - X_{3i},\ where\ X_{3i} = \frac{r_{5i}+jr_{6i}}{\sqrt{r_{5i}^2+r_{6i}^2}}\ and\ |X_{3i}| < 1$$

$$H_{3i}\left(Z\right) = \frac{X_{3i}\left(\frac{1}{X_{3i}}-Z\right)}{Z}$$

As $|X_{3i}| < 1, |\frac{1}{X_{3i}}| > 1$, system will have a zero outside the unit circle and a pole at origin and it's phase response will be having a transition for each zero of the system.

System III is formed by cascading 'K' subsystems of transfer function H_{3i} i.e. $C_K = \prod_{i=1}^{K} H_{3i}\left(Z\right)$. As this system has 'K' zeros, a total of 'K' transitions can be observed, each pole at origin introduces a linear phase w.r.t frequency.

Phase response of the individual subsystems H_{3i} and the system C_K are shown in figure 3 of the newsletter.

2.4 System IV:

Consider a system having transfer function

$$H_{4i}\left(Z\right) = \frac{1}{Z^{-1}-X_{4i}},\ where\ X_{4i} = \frac{r_{7i}+jr_{8i}}{\sqrt{r_{7i}^2+r_{8i}^2}}\ and\ |X_{4i}| < 1$$

$$H_{4i}\left(Z\right) = \frac{Z/X_{4i}}{\frac{1}{X_{4i}}-Z}$$

As $|X_{4i}| < 1, |\frac{1}{X_{4i}}| > 1$, It will have a pole outside the unit circle and a zero at origin and it's phase response will be having a transition for each pole.

System IV is formed by cascading 'K' systems of transfer function H_{4i} i.e. $D_K = \prod_{i=1}^{K} H_{4i}\left(Z\right)$. As this system has 'K' poles, a total of 'K' transitions can be observed, each zero at origin introduces a linear phase w.r.t frequency.

Phase response of the individual subsystems H_{4i} and the system D_K are shown in figure 4 of the newsletter.

3 Cascade of the Four systems taken two at a time:

When the systems I and II are cascaded, the resultant transfer function is $H(z) = \prod \frac{(z-x_{1i})}{(z-x_{2i})}$ i.e, resultant system has both poles and zeros within the unit circle. The phase response of the system is a difference of phase response of $\prod(z - x_{1i})$ and $\prod(z - x_{2i})$. Hence the phase response plot is a smooth curve w.r.t frequency.

Similar results can be observed for a cascade combination of systems I and III, systems II and IV, systems III and IV. Respective responses can be observed in plots a, b, e and f of figure 5 in the newsletter.

When the systems I and IV are combined, the resultant system transfer function is $H(z) = \prod \frac{(z)}{(x_{4i})} \frac{(z-x_{1i})}{(-z+\frac{1}{x_{4i}})}$.The resultant system phase response has an additional phase shift of 180 degree along with the difference in phases of $(z - x_{1i})$ and $(z - \frac{1}{x_{4i}})$. The combined phase response plot will have all the phase transitions that are exhibited by the individual systems.

Similar results can be observed when system II and III are cascaded. It can be observed in the plots c and d of figure 5 in the newsletter.