Stochastic Processes - Video course

COURSE OUTLINE

Probability Review and Introduction to Stochastic Processes (SPs): Probability spaces, random variables and probability distributions, expectations, transforms and generating functions, convergence, LLNs, CLT. Definition, examples and classification of random processes according to state space and parameter space.

Discrete and Continuous-time Markov Chains (MCs): Transition probability matrix, Chapman-Kolmogorov equations; n-step transition and limiting probabilities, ergodicity, stationary distribution, random walk and gambler's ruin problem, applications of DTMCs. Kolmogorov differential equations for CTMCs, infinitesimal generator, Poisson and birth-death processes, Applications to queueing theory, inventory analysis, communication networks. and finance.

Brownian Motion: Wiener process as a limit of random walk; first -passage time and other problems, applications to finance.

Branching Processes: Definition and examples branching processes, probability generating function, mean and variance, Galton-Watson brancing process, probability of extinction.

Renewal Processes: Renewal function and its properties, renewal theorems, cost/rewards associated with renewals, Markov renewal and regenerative processes, applications.

Stationary Processes: Weakly stationary and strongly stationary processes, moving average and auto regressive processes.

Martingales: Conditional expectations, definition and examples of martingales, inequality, convergence and smoothing properties, applications in finance.

COURSE DETAIL

Module	Topics and Contents	Lectures
1	**Probability Theory Refresher:** Axiomatic construction of probability spaces, random variables and vectors, probability distributions, functions of random variables; mathematical expectations, transforms and generating functions, modes of convergence of sequences of random variables, laws of large numbers, central limit theorem.	3
2	**Introduction to Stochastic Processes (SPs):** Definition and examples of SPs, classification of random processes according to state space and parameter space, types of SPs, elementary problems.	4
3	**Discrete-time Markov Chains (MCs):** Definition and examples of MCs, transition probability matrix, Chapman-Kolmogorov equations; calculation of n-step transition probabilities, limiting probabilities, classification of states, ergodicity, stationary distribution, transient MC; random walk and gambler's ruin problem, applications.	8
4	**Continuous-time Markov Chains (MCs):** Kolmogorov-Feller differential equations, infinitesimal generator, Poisson process, birth-death process, Applications to queueing theory, inventory analysis, communication networks, finance and biology.	7

NPTEL

http://nptel.iitm.ac.in

Mathematics

Pre-requisites:

Probability Theory and Calculus

Additional Reading:

1. P. G. Hoel, S. C. Port and C. J. Stone, Introduction to Probability Theory, Universal Book Stall, 2000.
2. S. M. Ross, An Introduction to Probability Models, 9th Edition, Academic Press, 2006.

Coordinators:

Dr. S. Dharmaraja
Department of Mathematics, IIT Delhi

5	**Brownian Motion:** Wiener process as a limit of random walk; first -passage time and other problems, applications to finance.	4
6	**Branching Processes:** Definition and examples branching processes, probability generating function, mean and variance, Galton-Watson branching process, probability of extinction.	4
7	**Renewal Processes:** Renewal function and its properties, elementary and key renewal theorems, cost/rewards associated with renewals, Markov renewal and regenerative processes, applications.	4
8	**Stationary Processes:** Weakly stationary and strongly stationary processes, moving average and auto regressive processes.	4
9	**Martingales:** Conditional expectations, definition and examples of martingales, inequality, convergence and smoothing properties, applications in finance.	4

References:

1. J. Medhi, Stochastic Processes, 3rd Edition, New Age International, 2009.
2. S.M. Ross, Stochastic Processes, 2nd Edition, Wiley, 1996 (WSE Edition).
3. G. R. Grimmett and D. R. Stirzaker, Probability and Random Processes, 3rd Edition, Oxford University Press, 2001.
4. H.M. Taylor and S. Karlin, An Introduction to Stochastic Modeling, 3rd Edition, Academic Press, New York, 1998.